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Abstracts of talks

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Talks

On the fundamental domain of tetrahedral spherical space forms

Ana Paula Tremura Galves

The topological spherical space forms problem is the study of fixed-point free actions of finite groups on spheres. Equivalently, it is the study of space forms.

The earliest examples of space forms are the Clifford-Klein manifolds. A Clifford-Klein manifold is a complete Riemannian manifold with constant sectional curvature equal to $+1$. They are of the form S^{4n-1}/G where G is a finite group acting freely and orthogonally on the sphere $4n-1$-dimensional (S^{4n-1}). The classification of Clifford-Klein manifolds is thus a completely algebraic question in group representation theory. A complete solution was given by J. Wolf [3].

We denote by P_{24} the binary tetrahedral group of order 24. This is the group with three generators and presentation $P_{24} = \langle x, y, z \,|\, x^2 = (xy)^2 = y^2, zxz^{-1} = y, zyz^{-1} = xy, xyx^{-1} = y^{-1}, z^3 = x^4 = 1\rangle$ that acts freely on the odd dimension spheres.

The main purpose of this work is to describe a fundamental domain of the spherical space forms which fundamental group is the binary tetrahedral group, that we call *tetrahedral spherical space forms* and we denote by $\mathcal{P} = S^{4n-1}/P_{24}$.

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Some results on pairs of involutions

Elizabeth Salazar Flores

In this work we present the study of pairs (φ_1, φ_2) of involutions on $(\mathbb{R}^n, 0)$ associated with divergent diagrams of folds $(f_1, f_2) : (\mathbb{R}^n, 0) \to (\mathbb{R}^n \times \mathbb{R}^n, 0)$. We also obtain normal forms for the pairs (φ_1, φ_2) and, consequently, normal forms for the corresponding divergent diagram of folds (f_1, f_2), when the involutions are linear and transversal. In addition, we present a discussion of our results to the study of a class of discrete reversible dynamical systems. Finally, we give a characterization of the group generated by any pair of linear involutions.

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Normal Form of reversible-equivariant vector fields

Iris de Oliveira Zeli

In a dynamical system, the presence of symmetries or reversing symmetries leads to the occurrence of multiple solutions: symmetries take trajectories in trajectories, preserving the direction with time, whereas reversing symmetries take trajectories in trajectories, reversing direction with time. When both occur simultaneously, the system is called reversible-equivariant and all these objects have structure of group, the group of symmetries and reversing symmetries of the system, denoted by Γ. This fact implies the existence of a normal subgroup of index 2, formed by symmetries of Γ and denoted by Γ_+. Thus, the mathematical formulation for this theory is made through the group representation theory. The study begins by considering a group homomorphism

$$\sigma : \Gamma \to \mathbf{Z}_2, \tag{1}$$

where $\mathbf{Z}_2$ is the multiplicative group $\{-1, 1\}$ e $\Gamma_+ = \ker(\sigma)$. A vector field $g : V \to V$ is called Γ-reversible-equivariant if

$$g(\gamma x) = \sigma(\gamma)\gamma g(x),\ \forall \gamma \in \Gamma,\ x \in V. \tag{2}$$

In the study of various dynamic phenomena, a method of great interest is to pass the vector field for the normal form Belitskii. In fact, this has been a very efficient tool for the study of bifurcations, the occurrence of periodic solutions, limit cycles and other local and global phenomena. In this work, we deduce how to obtain this normal form preserving all symmetries and reversing symmetries of system. The method is based on the representation theory of algebraic groups and invariant theory.

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Holonomy Invariant Measure

Jorge Crisostomo

Consider a differentiable manifold M, a foliation $\mathcal{F}$ on M and μ a measure defined in the space transverse to the foliation $\mathcal{F}$ which is invariant "by the flow of the leaves," this measure is called holonomy invariant transverse measure. The existence of such measure is important and useful to understand the dynamics of foliations. In this lecture, we present the sufficient conditions for the existence of such measures, for which we introduce the definition of a leaf growth. The growth of a leaf will be defined in order algebraic and geometric, and we will show that these two notions coincide. Our goal is to prove the following existence theorem of J. F. Plante.

Theorem: Let $\mathcal{F}$ be a codimension k foliation of a compact manifold M. If L is a leaf of $\mathcal{F}$ having non-exponential growth then there exists a non-trivial holonomy invariant measure for $\mathcal{F}$ which is finite on compact sets and wich has support contained in the closure of L.

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A study on the equation of Hénon

Maribel Bravo Quispe

We will make a quantitative and qualitative study of positive solutions for the following Dirichlet problem for the Hénon equation,

$$(P)\begin{cases} -\Delta u &= |x|^{\alpha}|u|^{p-2}u \quad in \quad B, \\ u &= 0 \quad on \quad \partial B, \end{cases}$$

where B is the unit open ball of $\mathbb{R}^N$ centered at zero and $\alpha > 0$. We will show that for $p \geq 2^*_\alpha$, where

$$2^*_\alpha = \begin{cases} \frac{2(N+\alpha)}{N-2}; & N > 2, \\ \infty; & N = 1, 2, \end{cases} \qquad 2^* = \begin{cases} \frac{2N}{N-2}; & N > 2, \\ \infty; & N = 1, 2, \end{cases}$$

the problem does not have nontrivial solutions. In counterpart, for $1 < p < 2^*_\alpha$ with $p \neq 1$, the existence of radial positive solutions will be guaranteed. Moreover, the uniqueness of a positive solution will be guaranteed as long as $1 < p < 2$. In addition we will present results on the existence of Ground State solutions when $2 < p < 2^*$ (critical exponent of Sobolev). In this interval we will show that any Ground state solution exhibits the Foliated Schwarz symmetry and, in case α is sufficiently large, we will show that the no Ground state is radially symmetric. To complete our study, we will present results on the existence of multiple positive solutions. In particular, we will show that (P) has $\left[\frac{N}{2}\right] - 1$ non radial solutions in case $N \geq 4$, $\alpha > N + 2$ and $p \in (2, 2(N-1)/(N-3))$.

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Skew product semiflows and Morse decomposition

Matheus Cheque Bortolan

This lecture is dedicated to investigate the dynamics of non-autonomous differential equations. We emphasize the fact that the asymptotics of non-autonomous problems are in fact described by a set (usually infinite) of processes each of them corresponding to a limiting non-autonomous differential equation for which one can define an evolution process. We then study the Morse decomposition of these non-autonomous problems

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On the cohomological degree of maps between generalized manifolds

Norbil Cordova

Generalized manifolds are a class of spaces which reflect many of the local and global homology properties of the topological manifolds. They were introduced by Čech and Lefschetz and were studied in detail in R.L. Wilder's book, *topology of Manifolds*. Generalized manifolds show up naturally in topological problems in the theory of transformation groups and in some other parts of topology.

The notion of degree, which was introduced by Brouwer, is an important object in algebraic topology to the study of continuous maps. After some time, based on higher (co)homologies of manifolds, a fairly complete (co)homological degree theory was constructed and successfully applied, first for continuous maps of compact manifolds only, then for proper continuous maps of arbitrary orientable topological manifolds, and later including manifolds with boundaries.

We are concerned in studying the cohomological degree of continuous maps between generalized manifolds, including generalized manifolds with boundaries. Here the degree is constructed using results of the cohomology and homology theory of topological spaces with coefficients in a sheaf. The importance of working with this type of coefficients is because the generalized manifolds are defined in terms of the sheaves theory and moreover the sheaves allow that the Poincaré Duality be satisfied for these spaces not necessarialy orientable.

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Fractional Differential Equations: A Novel Study in Banach Spaces

Paulo Mendes de Carvalho Neto

The fractional calculus can be considered in many ways, a novel topic, once it is only during the last thirty years that it has been the subject of specialized conferences and treatises. Everything has begun with the important applications discovered in numerous diverse and widespread fields in science, engineering and finance, see [1, 2, 3, 4, 5] among others.

Hence, motivated by the huge success of this applications we start trying to answer unsolved questions of this theory. We want to deal with the abstract fractional Cauchy problems of order $\alpha \in (0,1)$, i.e.,

$$\begin{cases} cD_t^{\alpha} u(t) = Au(t) + f(t, u(t)), \quad t > 0 \\ u(0) = u_0 \in X, \end{cases}$$

where X is a Banach space, $A : D(A) \subset X \to X$ is a sectorial operator, cD_t^{α} is Caputo's fractional derivative and $f : \mathbb{R}^+ \times X \to X$ is a suitable function.

In general, we answer questions that were not completely studied: for instance, we analyze the existence of local mild solutions for the problem, and its possible continuation to a maximal interval of existence and using some general comparison results in Banach spaces we guarantee the global uniqueness and existence of solution. The case of critical nonlinearities and corresponding regular mild solutions is also studied.

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Properties of the Fiber Cone

Pedro Henrique Apoliano Albuquerque Lima

Given a Noetherian local ring $(R, \mathfrak{m})$ and an ideal I in R, there is a natural filtration $... \subseteq I^2 \subseteq I \subseteq R$, called adic-filtration. One may construct a graded algebra $F(I) = \oplus_{n \geq 0} I^n/\mathfrak{m}I^n$, called fiber cone or special ring. Moreover, naturally it is possible to generalize this algebra by using any filtration of ideals $\mathfrak{F} : ... \subseteq I_2 \subseteq I_1 \subseteq R$. It is denoted by $F(\mathfrak{F})$. The goal of this talk is to speak about the Gorenstein property of $F(\mathfrak{F})$ and its Castelnuovo-Mumford regularity.

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SRB Measures and Uniqueness of SRB measures for transitive diffeomorphisms on surfaces

Pouya Mehdipour

First I will talk about the SRB measures and the importance of their existence. Then through giving a description of ergodic components of SRB measures in terms of ergodic homoclinic classes associated to hyperbolic periodic points, will talk about the results of [HHTU] about the uniqueness of SRB measures for transitive diffeomorphisms.

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Variational characterization of the Fučík spectrum for systems

Rafael Antnio Rossato

We study the Fučík Spectrum for the case of a coupled system of two elliptic equations, in order to obtain a variational characterization for regions above the first nontrivial curve. For this purpose we apply a classical Linking Theorem using sets obtained by a suitable deformation of the eigenspaces.

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Atypical values and Morse-Sard type theorem

Luis Renato Gonçalves Dias

We describe the relation between four different types of regularity conditions which have been used in the literature in order to control the asymptotic behaviour of semi-algebraic mappings. We prove a new Morse-Sard type theorem for the asymptotic critical values of semi-algebraic mappings. For polynomial mappings, we also present some algebraic characterizations of regularity at infinity.

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Free structures in some division rings

Renato Fehlberg Junior

The existence of free structures in a division ring is related to how complex its structure is. In 1984, Makar-Limanov conjectured that if a division ring D is finitely generated and infinite dimensional over its center (a field) k then D contains a noncommutative free k-algebra over 2 generators. I will discuss interesting known results, and I will show, using the techniques created by Makar-Limanov, how we gave a positive answer when the division ring is the fraction field of the skew polynomial ring $L[t, \sigma]$, where L is the function field of an abelian variety.

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New characterization of trivial maps in 3-dimensional real Milnor fibers

Taciana Oliveira Souza

In the book "Singular points of complex hypersurfaces", [1], John W. Milnor studied singular points on hypersurfaces introducing a locally trivial fiber bundle, called *the Milnor fibration*, associated to each singular point. He shows there are such locally trivial fiber bundle for germs of holomorphic maps and for germs of real analytic maps. We are interested specially in the real case, where Milnor considers a real polynomial mapping $f : (\mathbb{R}^n, 0) \to (\mathbb{R}^p, 0)$. Milnor proposed to call the singularity *trivial* if the fiber of the Milnor fibration associated is diffeomorphic to the disk, and he asked [1, p.100]:

"For which dimensions $n \geq p \geq 2$ do non-trivial examples exist?"

Church and Lamotke largely answered this question in [2]. In [3], we extend the characterization of trivial map germs for the real Milnor fibration started by Church and Lamotke, our main result covers all cases on the 3-dimensional real Milnor fibers.

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Universal positive definite kernels

Victor S. Barbosa

Positive definite kernels appear as important tools in many branches of mathematics such as linear algebra, functional analysis, approximation theory, just to mention a few (see [2]). In particular, the modern learning theory was built upon some key properties of the positive definite kernels (see [3]).

Perhaps, the most important property a positive definite kernel $K : E \times E \to \mathbb{C}$, where E is a topological space, has is this (see [1]): there exists a Hilbert space $(\mathcal{W}, \langle\cdot,\cdot\rangle)$ and a mapping $\phi : E \to \mathcal{W}$ such that

$$K(x,y) = \langle \phi(x), \phi(y) \rangle, \quad x, y \in E. \tag{1}$$

The function ϕ is usually called a *feature map* of K while $\mathcal{W}$ is called the *feature space*. It is well known that $\mathcal{W}$ and ϕ are not unique.

A continuous kernel K as above has *the universal property for approximation* (that is, K is universal) if for every $\varepsilon > 0$ and every compact subset X of E the following property holds: if f is a continuous function on X, there exists a function g in the closure $G_K(X)$ of $[\{K(\cdot, y) : y \in X\}]$ in $C(X)$, the space of all complex valued continuous functions on X, endowed with its uniform norm $\|\cdot\|_\infty$, so that $\|f - g\|_\infty < \varepsilon$. Since $G_K(X)$ is closed in $C(X)$, the requirement above corresponds to $G_K(X) = C(X)$.

Let K be a continuous kernel as above and consider an orthonormal basis $\mathcal{B}$ of $\mathcal{W}$ (the existence of that is guaranteed in [4, p.144]). The feature map ϕ is *universal* if for every $\varepsilon > 0$, every compact subset X of E and every f in $C(X)$, there exists a function g in the closure $\phi(\mathcal{B})$ of $[\{\langle \phi(\cdot), v\rangle : v \in \mathcal{B}\}]$

in $C(X)$ such that $\|f-g\|_\infty < \varepsilon$. Here, the closeness of $\phi(\mathcal{B})$ in $C(X)$, reveals that the requirement above corresponds to $\phi(\mathcal{B}) = C(X)$.

The intended goal in this talk is to prove the following relationship between the two concepts introduced above: a continuous and positive definite kernel $K : E \times E \to \mathbb{C}$ is universal if and only if the corresponding feature map ϕ is so.

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